A Rapid Method of Calculating N-Factors for Estimating Transition Position

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ABSTRACT

Transition estimation using the E-to-N approach involves evaluating large numbers of eigenvalues of the linear perturbation equations. These eigenvalues provide the amplification rates of the modes of various frequencies and angles of orientation. The number of evaluations required to determine the N-factors can be large, especially when cross-flow modes over a leading edge are important. But since the variation in the values of the amplification rates varies relatively slowly and smoothly with the parameters involved in the problem, it seems that a fresh evaluation of the eigenvalue by solving the governing equations for every mode is somewhat of an overkill. Here we explore the idea that the behaviour of the eigenvalues can be described by some compact expansion. An appropriate functional form has been found that describes the eigenvalues in terms of the controlling parameters and the necessary coefficients evaluated so that predictions of N-factors can be made for two-dimensional compressible boundary layers very rapidly indeed. The N-factor curves on a typical aerofoil boundary layer can be determined roughly 2500 times as fast as by the use of a direct solver. The accuracy is well within the requirements of the E-to-N prediction method. Extension to three-dimensional flows is currently being undertaken.

1.0 INTRODUCTION

It has become increasing important to be able to estimate the most likely position of the laminar-to-turbulent transition in boundary layers on aerodynamic surfaces. It is sometimes thought that this process is only of importance when attempting to realize the maximum amount of laminar flow on an aerofoil. But the need for transition estimation arises in many other problems concerned with aviation.

If one wants to make estimates of wing drag, for performance calculations say, it is essential to have a reliable way of predicting the correct position for the start of the turbulent boundary layer to enable an appropriate turbulence model to be invoked.

Wind tunnel tests are often carried out at reduced Reynolds numbers and it is vital to be able to fix artificially the transition position on the model at the most appropriate place so as to replicate the behaviour at full scale.

Full numerical calculations of the flow over a wing often involve an outer Euler solution that is coupled to a boundary layer code in some iterative way. It is therefore necessary to be able to establish the position of transition so that the whole boundary layer can be modelled, and this may have to be done within the iteration cycle, and so requires a very rapid scheme for estimating where the turbulent boundary layer begins.

There are various ways of making these estimates, from very simple quick correlation methods, linear amplification codes that provide N-factors, PSE codes that can be nonlinear and full DNS evaluations. All approaches have there place, but the linear E-N method proposed almost 50 years ago by Van Ingen and independently by A M O Smith is the most commonly used industrial tool. The basic idea behind the method is that linear stability theory can provide the degree of amplification of unstable travelling waves as they progress downstream. It is then assumed, not unreasonably, that when the amplitude of the waves is sufficiently large the flow will somehow become turbulent. The environment will influence the magnitude of the perturbations at inception that occurs in the region of neutral stability. It turns out that transition takes place when the amplification of the waves reaches some threshold that depends on this level of environmental excitation. In a quiet environment provided by flying at altitude the critical amplification, or N-factor, is reasonably constant at a value of around 9, in a noisy wind tunnel the critical value of N will be reduced to possibly 6 or 7.

The theory was originally based on temporal stability theory and used the phase velocity to provide spatial growth rates. Now-a-days a spatial quasi-parallel formulation is used, and compressibility and three-dimensional modes can be included. Some corrections to the value of critical "N-factors" for certain types of roughness can also be made. Similarly, I believe that suction effects can be included in a relatively simple way. The N-factor method provides Industry with a plausible way of estimating the position of transition. Clearly the method is quite simplistic in its approach and as it relies on empirical correlations, but it is generally fast enough and reliable enough to be useful.

In 1975 I was trying to calculate the wavepacket created by an impulsive point excitation. This involved calculating large numbers of eigenmodes from the Orr-Sommerfeld equation, covering a range of frequencies, spanwise wavenumbers and Reynolds numbers. Although only about 10,000 eigenvalues were needed this, at the time, presented an enormous computation effort. Each eigenmode took about 5 seconds to evaluate and therefore some 12 hours of mainframe were needed to complete the task. A faster method of calculating modes was required.

The Orr-Sommerfeld equation has regular coefficients and the characteristic function linking the parameters together should therefore also be regular without any singularities. The functional behaviour of the link between the dominant eigenvalue, α, and the parameters ω and R will also be well behaved in finite domains. α is the wavenumber, ω the frequency parameter and R the displacement thickness Reynolds number. Frequency, omega and Reynolds number will also be well behaved in finite domains. There can be branch points, but so far these have not caused any problems. It seemed reasonable, therefore, to attempt to define the functional form for the Blasius mean flow as a power series expansion. Once the coefficients had been evaluated it was a simple and quick operation to calculate an eigenmode. Unfortunately the series did not always converge. This was remedied by invoking a Shanks [2] summation procedure. Then eigenvalues correct to 10^{-7} could be obtained roughly 1000 times as fast as by the direct solution of the equations. This method was presented at the Agard meeting in Copenhagen in 1977 with the suggestion that the approach could be applied to a range of profiles so as to construct a fast E-to-N method [1]. I was told that this idea was unnecessary because there were computers 100 times faster than KDF9 used by me at the National Physical Laboratory.

Nearly 30 years on we have machines that are, of course, even faster, but we now want transition estimates for swept wings over a wide range of conditions. The numbers of linear eigenvalues required has increased so much that again we need to look for a faster method of solution.

The paper discusses some recent efforts to build a practical code that can be used on swept wings in compressible flow to provide rapid transition estimates.

2.0 THEORETICAL

The earlier method that used series expansions was difficult to use because the calculation of the series coefficients was time consuming. It also seemed more sensible to set up a Pade approximant scheme to define the eigenvalue as then the need for the Shanks convergence algorithm would be unnecessary. Although this scheme worked well enough the formation of the Pade tables was complex. Finally, a simpler scheme that used a ratio of polynomials in the Pade form to describe the relationships between the parameters was again employed, but the coefficients were found by more directly.

$$\alpha = F(R^{1/2}, 1/\omega)_,$$

where α is the wavenumber, ω the frequency parameter and R the Reynolds number.

A large number of points over the frequency~Reynolds number plane covering a space over which a solution was required was chosen as indicated in figure 1. At these randomly chosen points eigenvalues were evaluated using a normal Orr-Sommerfeld solver, and the coefficients of the Pade form were then fitted with minimum error to the data set. The best fit could be arranged to cover the domain of interest. Of course, there are a number choices as to the best functional form and the actual arrangement of the coefficients.

$$\alpha = \frac{\sum\sum A_{nm} R^{n/2}(1/\omega)^m}{\sum\sum B_{nm} R^{n/2}(1/\omega)^m},$$

where A_{nm} and B_{nm} are the coefficients that provide the best fit to the data.

Using the chosen functional form a series of test were carried out using various array dimensions. It was found that a triangular array of size 9 by 9, containing 109 coefficients provided a good compromise between accuracy and speed. As an example the eigenvalues for the Blasius profile calculated by this scheme are shown on figure 2. There is no point in also showing the stability loop computed directly from the perturbation equations as the differences between these two calculations are negligible, but the differences plotted as an error can be displayed over the same region as a contour display. Figure 3 shows the error distribution for the large array involving 109 coefficients. Here they are within 1 part in 10-5 over the significant portion of the plane, and are better than this over the most amplified portion of the loop. Greater accuracy could be obtained with larger arrays, and faster evaluations could be created by smaller arrays with a consequent loss in accuracy. As an example of this we show error distribution for a small array of 6 by 6 containing 55 coefficients on figure 4. These errors are still probably small enough for N-factor estimation, but for the time being the work has concentrated on getting a system in operation and we have kept the large array.

2.1 Incompressible Flow

For the case of incompressible flows it seems that the boundary layer can adequately be defined by the similarity profiles of Falkner-Skan family. Real boundary layers are not precisely of this form as there are inevitably some residual spatially varying terms that prevent the equation separating, but these effects are usually small. With this assumption the boundary layer can be reduced to a one parameter family of profiles. By then calculating the required Pade Coefficients for a range of profile shapes one can construct a rapid N-factor scheme. It turned out that only 9 profiles designated by the shape parameter H were needed. Then data for the specified aerofoil could be used to create the amplification, or N-factor, curves for a range of frequencies. The pressure distribution and associated boundary layer parameters for a specific test case are shown on figure 5. The N-factor calculation for this data set obtained by Dera is

shown on figure 6. This can be compared with that obtained by carrying out a full Orr-Sommerfeld calculation on the profiles given by the Falkner-Skan family on figure 7. Finally, N-factor curves calculated by the Pade model is shown on figure 8. The differences between figures 6 and 8 are negligible, but the new method was some 2500 times faster. Other pressure distributions gave similar results.

2.2 Compressible Flow

Civil transport aircraft operate at speeds that require the estimation of N-factors up to Mach numbers of 1.5, military aircraft cover a much wider range. The current codes were mainly developed for civil transport vehicles. Similarity solutions of the compressible boundary layer equation do not exist in the same way that they do for incompressible flows. The standard formulation of the mean flow boundary layer generally follows that of Reshotko, but the quasi similarity formulation used contains a non-negligible RHS to the equation set. These terms prevent the separation of variables and are therefore always ignored, but this procedure can create significant deviations in the profile shapes, that create large errors on amplification factors and cannot be ignored. A much better formulation has been suggested by Horton [5]. He used total enthalpy in his formulation and, for reasons that I certainly don't understand, derived equation with much weaker terms on the RHS of the equations.

Comparisons of the velocity and temperature profiles obtained by the two different quasi-similarity methods with solution of the full boundary layer equations for power law Mach number variations with streamwise distance showed that the Horton formulation was very close to the "exact" solution whereas the formulation normally derived in the text books was quite poor.

In more realist pressure distributions there will still be unresolved terms on the RHS of the equations that have to be neglected. The stability of the compressible flows was evaluated using the equations derived by Mack cast in a form involving the freestream Mach number squared. A shooting process was employed and satisfactory agreement with some test cases validated the numerical scheme. The rapid scheme used for incompressible flow was again employed for a series of Mach numbers covering the range of interest. It turned out that as the stability behaviour was a relatively weak function of Mach number, only 5 values of Mach number squared were needed to cover the range required adequately. An interpolation scheme was then used to focus on the precise value of shape parameter and Mach number. This involved using a 9-point interpolation scheme. As in the incompressible case the two-dimensional Pade summation reduced to a one dimensional form once the Reynolds number and Mach number of a specific station has been fixed. Any number of frequencies could then be evaluated very rapidly.

A test of the scheme was again set up against data computed by Chris Atkin of Dera. We found that in most examples the N-factor evaluations were quite close to those obtained using the full boundary layer equations to produce the velocity and temperature profiles. The N-factor evaluation showed a similar increase in computational speed compared with the direct evaluation as had been obtained in the incompressible situation.

2.3 Three Dimensional Boundary Layers

Three-dimensional boundary layers that form on swept wings, especially in the region around the leading edge, require more complex descriptions of the profile shapes. Some work was carried out by Jiang [3] on a fully three-dimensional code that relied on the Falkner-Skan-Cooke family of velocity profiles. These one parameter profiles are not too bad an option when important three-dimensional modes occur downstream, but they are poor for the more important situation involving the flow around leading edges.

We have been looking at a two-parameter family of velocity profiles to cater for these cases. The profile family was generated by a correlation technique that used profiles generated by a full numerical boundary layer code that was run for a variety of pressure distributions that might be encountered on real wings.

The profiles were sometimes quite different from those generated by the F-S-C family. This may account for the fact that the earlier approach did not provide accurate amplification factors for some aerofoils.

The process is not fully operational yet as more work is required to create all the lookup tables etc.

3.0 DISCUSSION

The work so far carried out indicates that eigenvalue for N-factor estimation can be carried out very rapidly indeed. The accuracy is in most cases to within less that 0.5 in the value of N. There were, however, situations involving rapid changes in the pressure gradients where the profiles obtained by the similarity models did not fit well with the "exact" ones. Then N-factor estimation was less reliable. One has to consider the fact that the N-factor method of indicating transition location provides only a guide, in fact it is probably not worth calculating N-factors to any greater precision. The approach takes no account of the non-parallel nature of the boundary layer. This is known to modify amplification rates, especially of oblique waves. It is possible to make some modification of the coefficients in the tables to take account of boundary layer growth, but it is not yet clear whether or not this is worthwhile.

The balance between speed and accuracy has not been optimised, but it seems very likely that further increases in speed could be obtained while retaining acceptable accuracy.

4.0 CONCLUSIONS

A rapid method of calculating N-factors has been developed for two dimensional compressible flows.

It turns out to be some 2500 as fast as a direct solver, with errors of roughly 0.5 in the estimated value of N.

A three-dimensional code is being created, but is incomplete.

REFERENCES

[1]   Gaster, M. 1978 Series representation of the Eigenvalues of the Orr-Sommerfeld Equation. *Journal of Computational Physics*. **29,** No 2.

[2]   Shanks, D. 1955 Nonlinear transformations of divergent and slowly convergent sequences.   *J. Math. & Phys.*, **34**, 1-42.   [3] Gaster, M. 1997 Rapid Estimator of Eigenvalues for N-Factor Calculation. Contract report *DERA ASF/30800*.

[4]   Jiang, F. & Gaster, M. 1995 A rapid Scheme for Estimating Transition on Wings by Linear Stability Theory. *ICAS Conf. Proc.*, Irvine Calif.

[5]   Horton, H P 1997   Near-Similarity approximation for compressible laminar boundary layers on adiabatic walls. *J.Aerospace Engineering,*. **211**, part G5, pp. 285-294.

[6]   Gaster, M 2002 Progress report on the development of the Rapid N-Factor Estimator. QinetiQ *contract CU004-14519.*

[7]   Atkin, C J 2002 Approximate similarity solutions for compressible laminar boundary layers with non-zero pressure gradient. QinetiQ/FST/TR024962.

**A Rapid Method of Calculating
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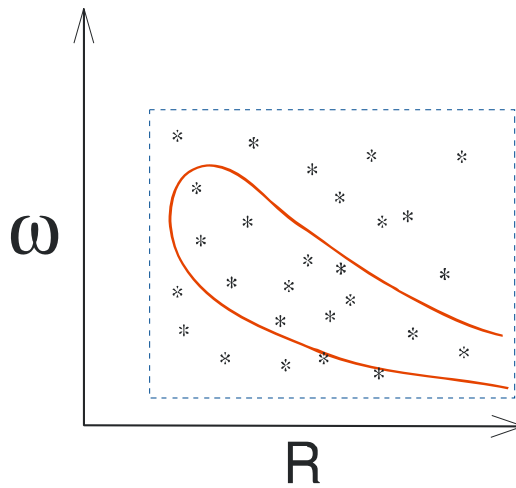
Figure 1 Random Eigenvalues over the stability loop
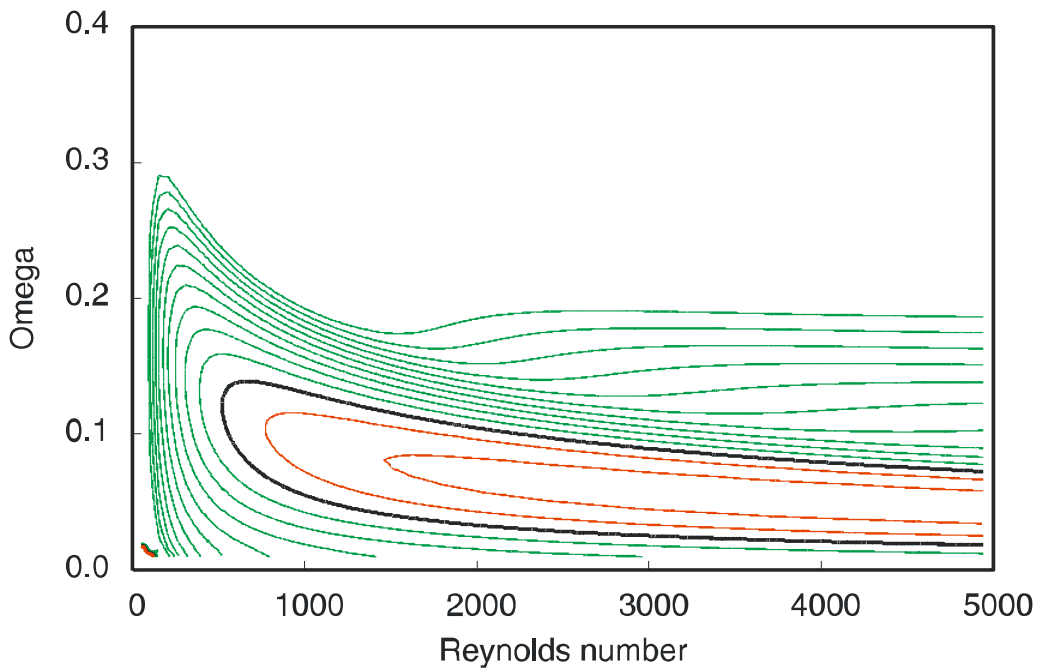
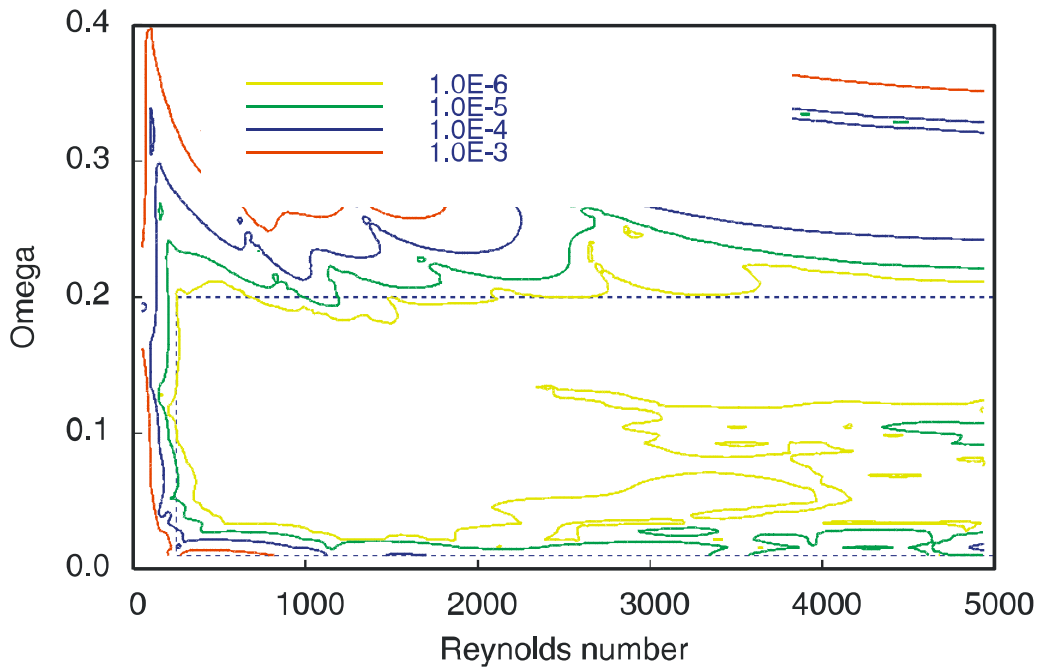**Figure 2 Amplification Contours for Spatial waves**

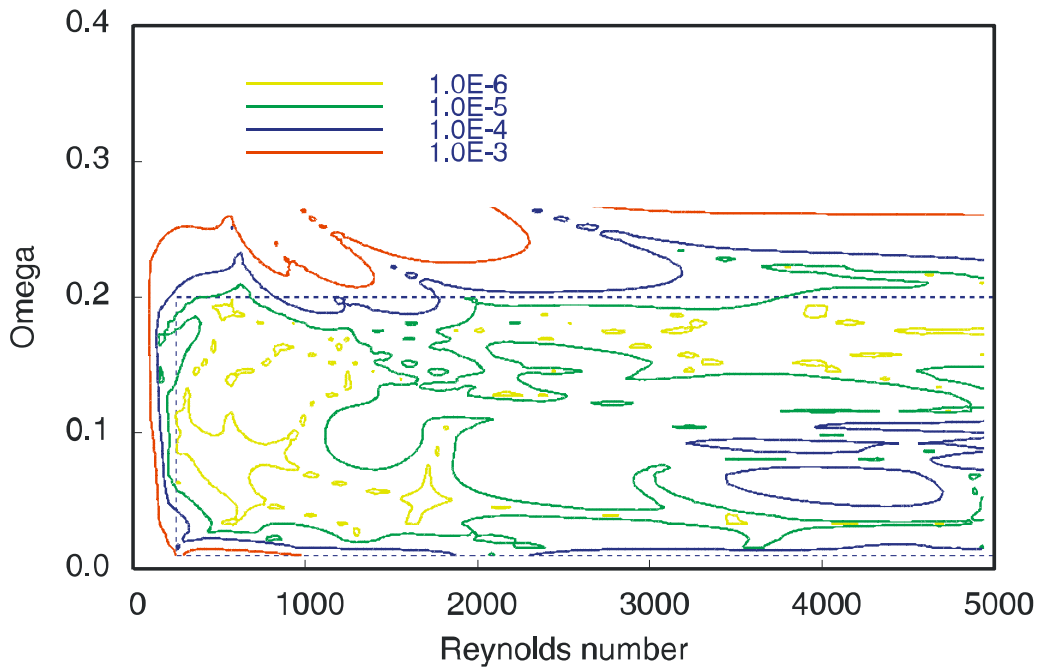**Figure 3 Error Contours for the 9 X 9 array**

Figure 4 Error Contours for the 6 X 6 array

Free-Stream Velocity = 50.0 M/s
Chord Length = 0.4305 M
Unit Reynolds number = 2.87 E6
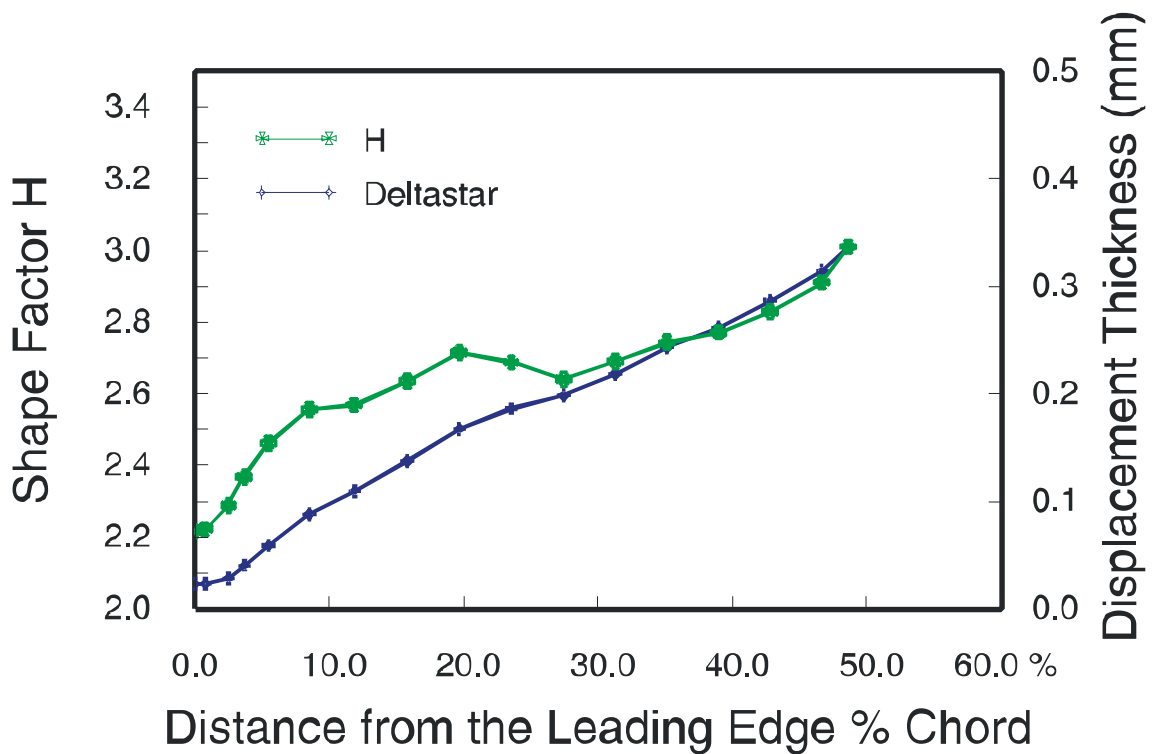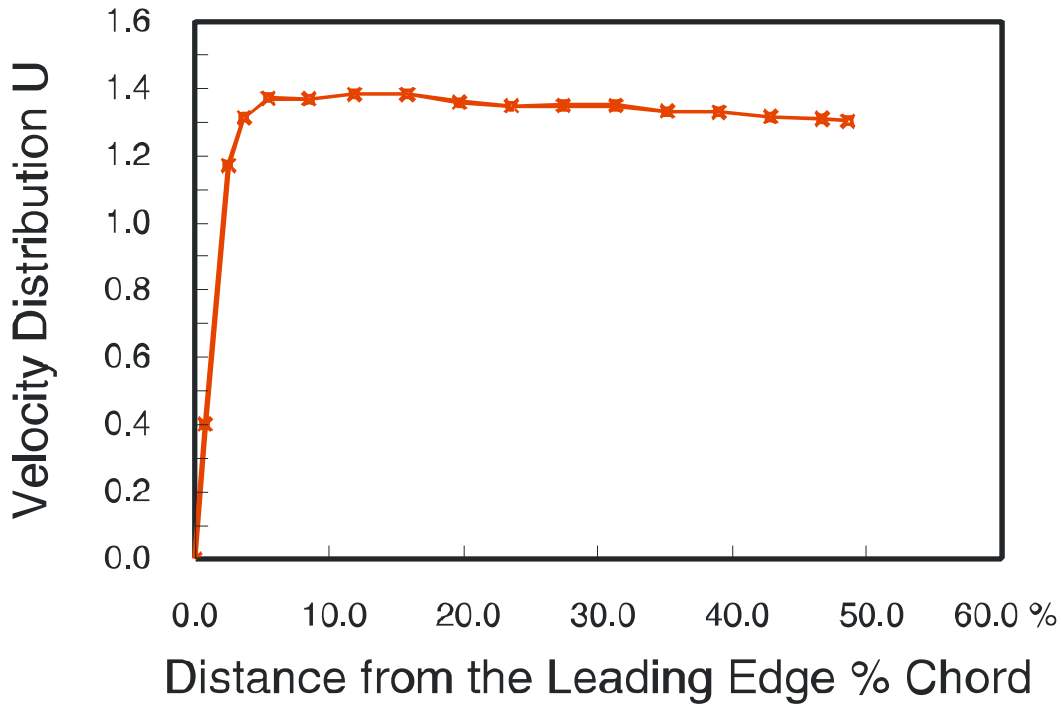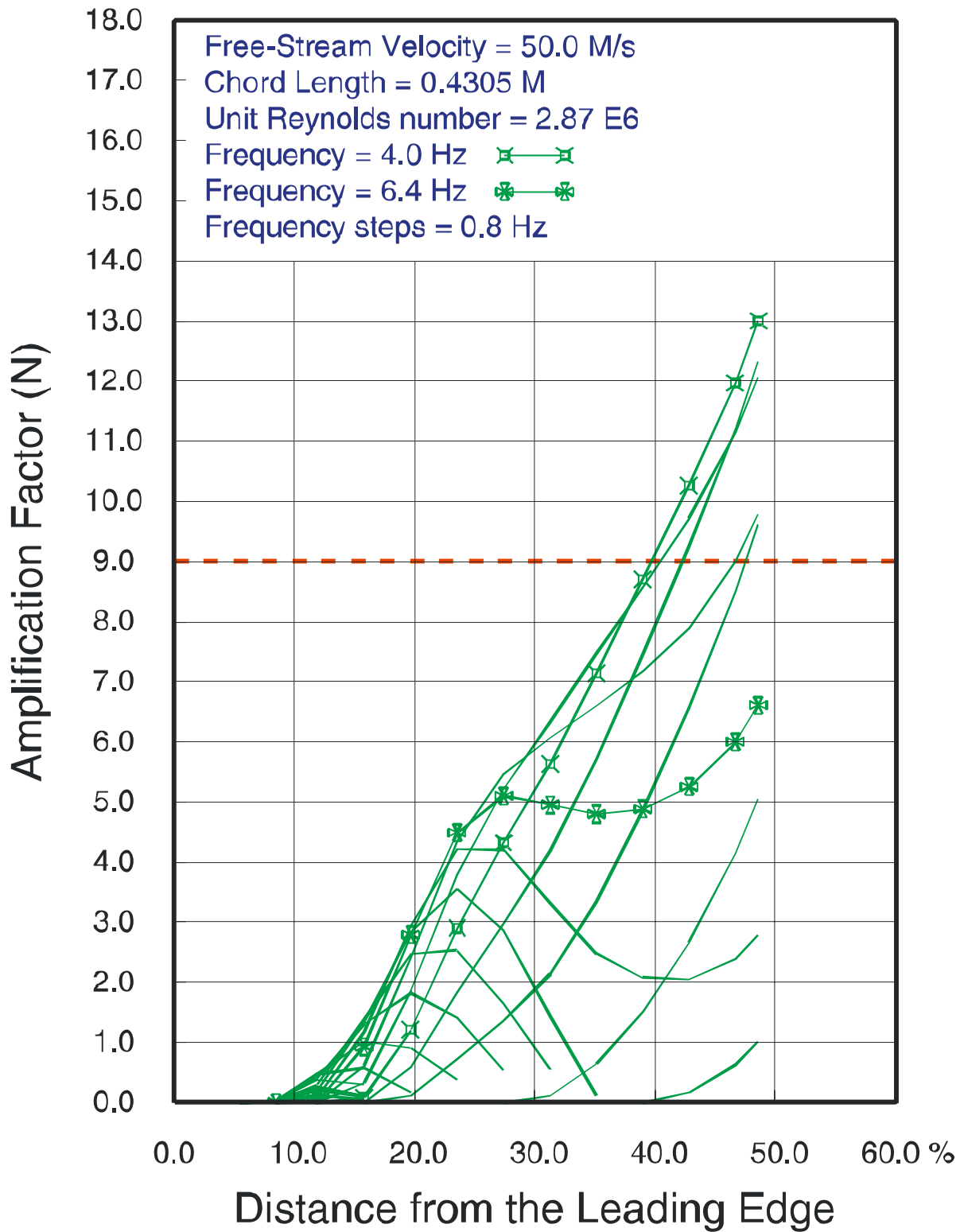

Figure 5 Boundary Layer Parameters for the Test Case
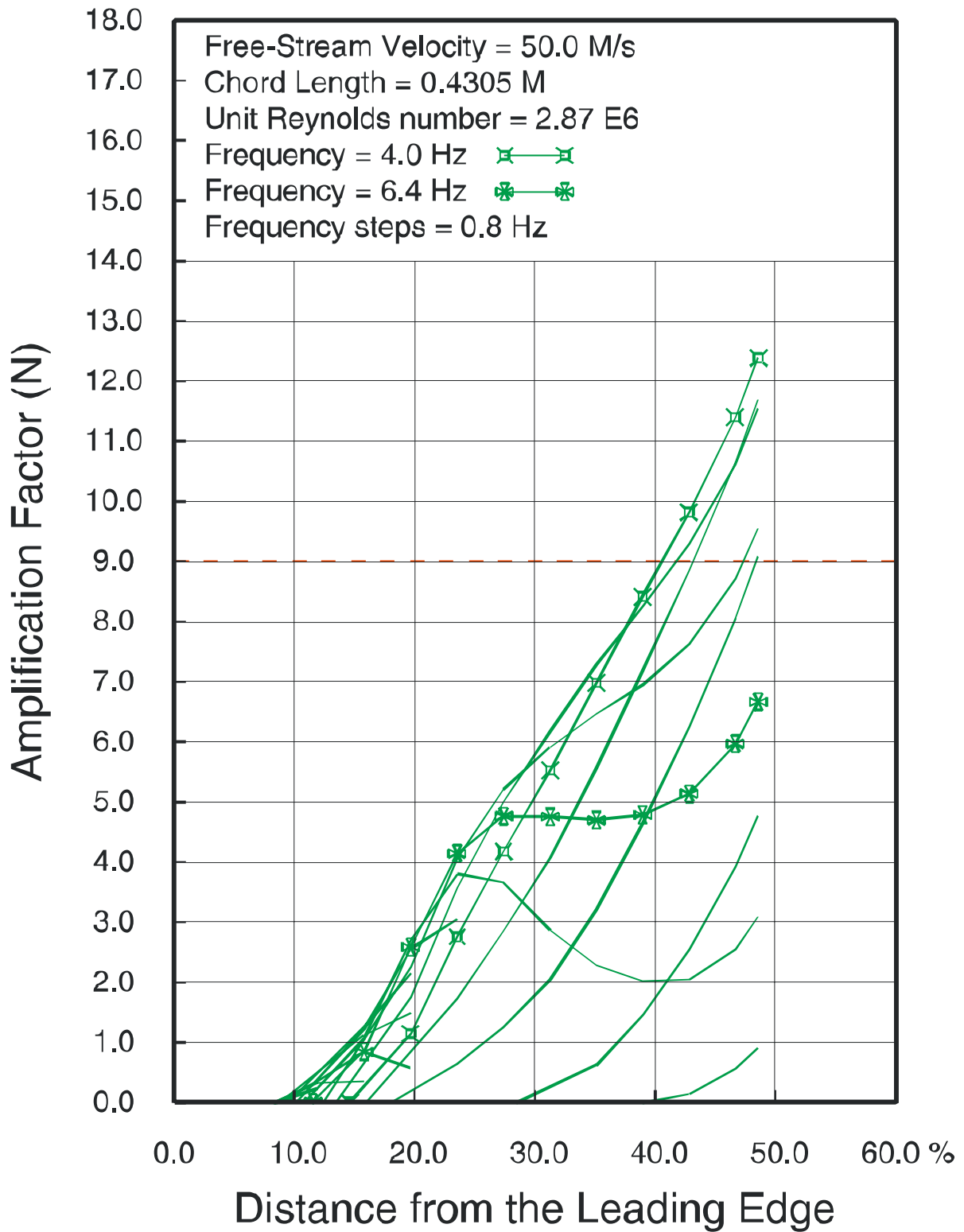
Figure 6 Amplification Factors for the true Boundary Layer

Figure 7 Amplification factors for Falkner-Skan profiles
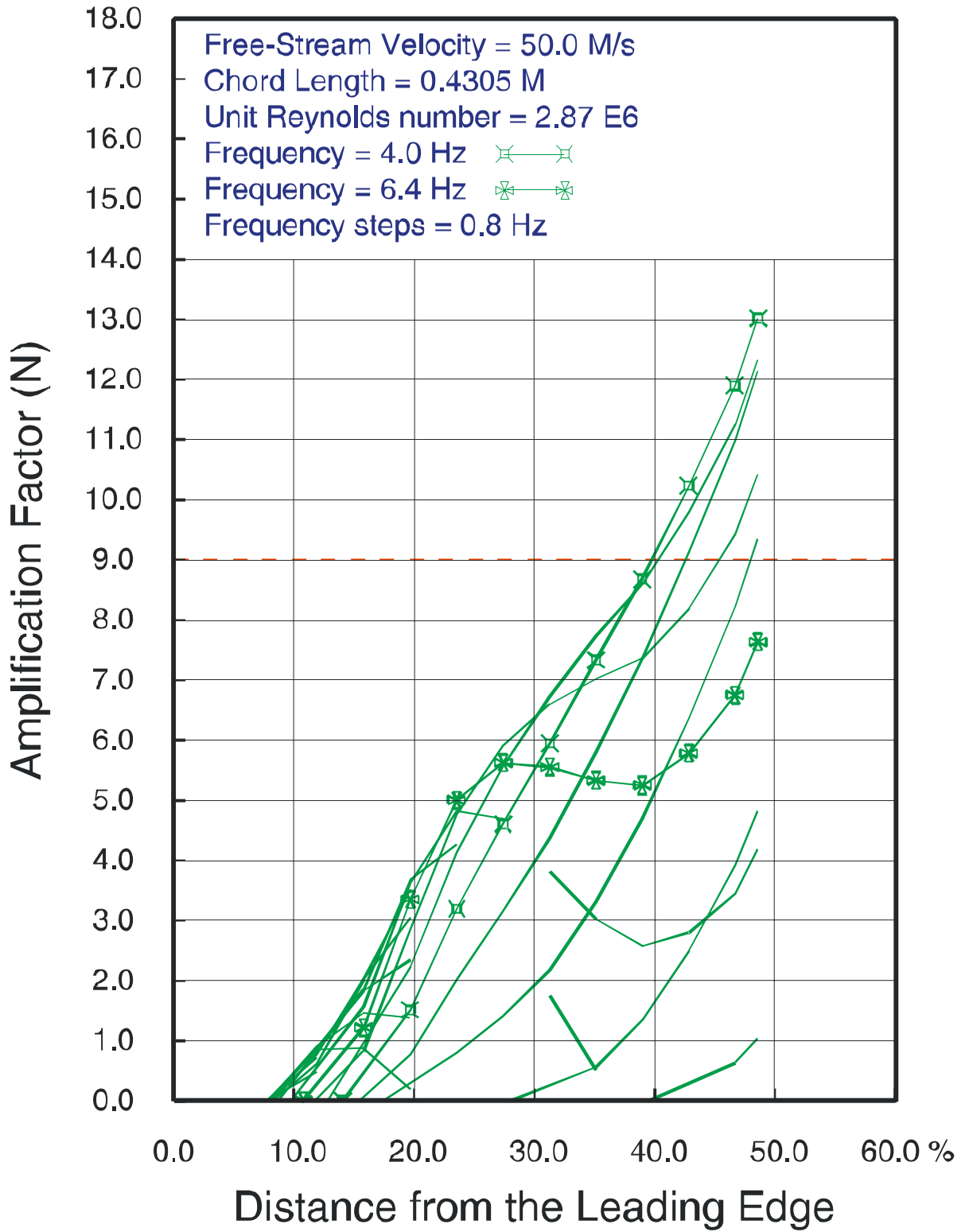
Figure 8 Amplification factors calculated by the 9X9 array

AVT-111 Specialists' Meeting on Enhancement of NATO Military Flight Vehicle
Performance by Management of Interacting Boundary Layer Transition and Separation

DISCUSSION

1. REFERENCE No. OF THE PAPER: 8
2. DISCUSSOR´S NAME: John Hourmouziadis
3. AUTHOR´S NAME: M. Gaster

QUESTION:
Would the general principle, not only the accelerated method, also apply to separated shear layers, i.e. to separation bubbles? Would it require an adapted velocity profile family?

AUTHOR´S REPLY:
Yes, but one could need to use a profile family that occur in separated regions. The Falkner-Skan family is not suitable.